Opening up the Weak Gravity Conjecture

- 2207.XXXX with Cesar Cota, Alessandro Mininno, and Max Wiesner
- Earlier work with Seung-Joo Lee, Wolfgange Lerche, Guglielmo Lockhart, and with Daniel Kläwer

See also Talk by A. Mininno on Thursday!

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Weak Gravity Conjecture

The **Weak Gravity Conjecture** is one of the pillars of the Swampland Program:

In every gauge theory coupled to gravity, there exists a particle with

$$\frac{g_{\rm YM}^2 q^2}{m^2} \geq \frac{g_{\rm YM}^2 Q^2}{M^2}\Big|_{\rm B.H.}$$

[Arkani-Hamed,Motl,Nicolis,Vafa'06]

Bottom-up motivation: Extremal black holes should decay

Tower WGC (tWGC): [Heidenreich,Reece,Rudelius] [Montero,Shiu,Soler]'16

[Andriolo,Junghans,Noumi,Shiu'18]

The super-extremal states must form an infinite tower.

Bottom-up motivation: [Heidenreich,Reece,Rudelius]'16-18

- Sufficient for consistency of the WGC under Kaluza-Klein reduction.
- But: The tower version is not strictly necesssary in presence of very super-extremal states, especially massless charged states.

Evidence for tWGC

Focus on *asymptotic tWGC*: in weak coupling limit $g_{\mathrm{YM}} \to 0$

$\Longleftrightarrow$ Swampland Distance Conjecture

1) Towers of BPS particles (e.g. 4d N=2, 5d N=1)

[Ooguri,Vafa'16] [Grimm,Palti,Valenzuela'18] [Gendler,Valenzuela'20] [Bastian,Grimm,Heisteeg'20] [Alim,Heidenreich,Rudelius'21] + many more

This includes (dual) KK towers in decompactification limits.
$\Longrightarrow$ Closed string U(1)

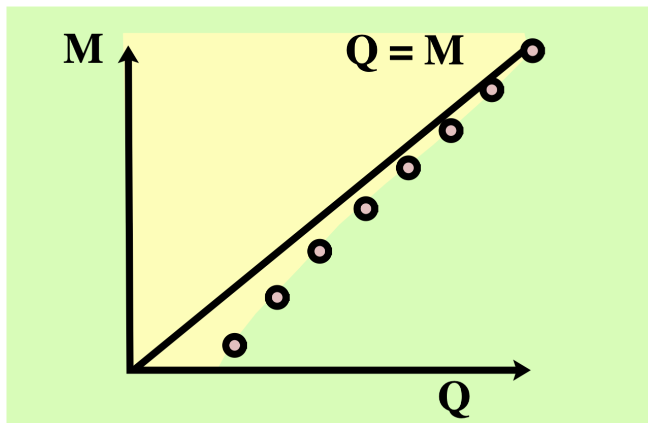
2) Perturbative heterotic string

Marginally super-extremal tower of string excitations from modularity

[Arkani-Hamed et al.'06]

[Heidenreich,Reece,Rudelius'16-'18]

[Montero,Shiu,Soler'16]



Pic: [Arkani-Hamed et al.'06]

Evidence for tWGC

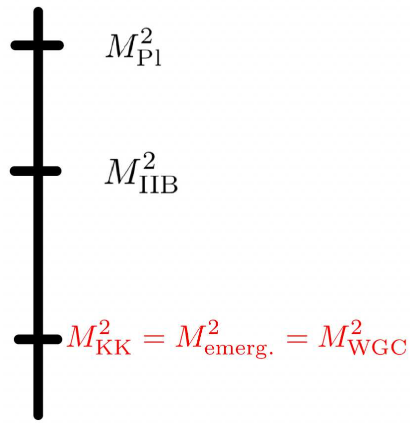
3) Open string theories:

F-theory in limits $g_{\rm YM} \to 0$ $\leftrightarrow$ Kähler moduli limits of base of elliptic fibration [Lee,Lerche,TW'18-19]

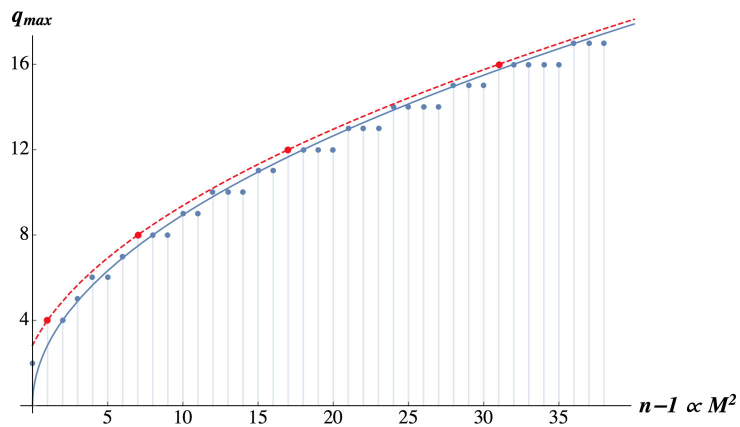
Case 1: Emergent String limits:

equi-dimensional limits with light emergent $critical$ (heterotic) string

$\Longrightarrow$ Marginally super-extremal tower of critical string excitations (not BPS!)



- 4d $N=1$ $\leftrightarrow$ quasi-Jacobi-ness [Lee,Lerche,Lockhart,TW'20][2]
- general non-perturbative situations if weak coupling limit for gauge group dual to perturbative heterotic gauge group
- explicit WGC relation, including 4d N=1 loop corrections [Kläwer,Lee,TW,Wiesner'20]



Weak Gravity Conjecture

Case 2: What about weak coupling limits not dual to heterotic limits?

Simplest example: F-theory on 4-fold over base

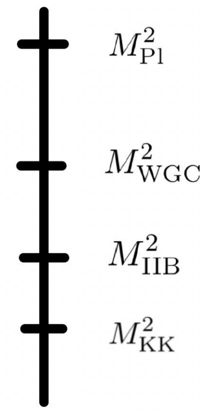
$$B_3 = \mathbb{P}^3 : \qquad H^{1,1}(B_3) = \langle H \rangle \quad \text{hyperplane class of } \mathbb{P}^3$$

Gauge theory from 7-brane on gauge divisor $D_H = H$

$$\frac{1}{g_{\rm YM}^2} \sim \mathcal{V}_H \sim v^2$$

$$\frac{M_{\rm IIB}^2}{M_{\rm Pl}^2} \sim \frac{1}{\mathcal{V}_{B_3}} \sim \frac{1}{v^3}$$

$$\frac{M_{\rm KK}^2}{M_{\rm Pl}^2} \sim \frac{1}{\mathcal{V}_{C_H}\mathcal{V}_{B_3}} \sim \frac{1}{v^4}$$



$\Longrightarrow$ weak coupling limit: $v \to \infty$

Tower WGC scale: $$\frac{M_{\rm WGC}^2}{M_{\rm Pl}^2} \sim g_{\rm YM}^2 \sim \frac{1}{v^2}$$

$\Longrightarrow$ coincides with tension of a string from a D3-brane on hyperplane curve

Weak Gravity Conjecture

Do excitations of the string furnish the asymptotic WGC tower?

cf.[Heidenreich,Reece,Rudelius'21][Kaya,Rudelius'22]

Two major differences to emergent string limit: [Cota,Mininno,TW,Wiesner'22]

1. The string is not a weakly coupled *critical* string.

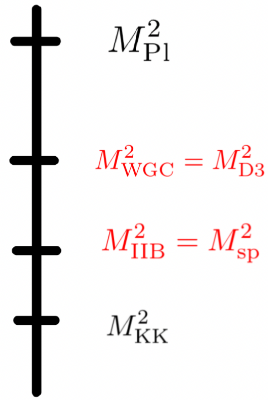
   Rather: non-critical EFT string [Lanza,Marchesano,Martucci,Valenzuela'20-21]

   $\Longrightarrow$ Exact WGC relation not satisfied without adhoc fudge factors!

2. The string does not set the species scale.

$$M_{\rm sp}^2 \sim M_{\rm IIB}^2 \ll M_{\rm WGC}^2$$

   $\Longrightarrow$ Do not expect the tWGC to hold.



In weak coupling limit:

Homogenous expansion of B_3 leads to 10d gravity theory coupled to an 8d defect gauge theory $\Longrightarrow$ Failure of tWGC is consistent!

This talk

General analysis of tWGC in the weak coupling limits for 4d N=1 gauge theories on 7-branes in F-theory

Main message: **[Cota,Mininno,TW,Wiesner - to appear]**

The tWGC holds asymptotically only

1. if the limit is towards $g_{\rm YM} \to 0$ and weakly coupled EFT string
2. if the species scale is set by the weakly coupled EFT string

$$M_{\rm sp}^2 \stackrel{!}{=} M_{\rm sp,string}^2 \sim T_{\rm string} \sim M_{\rm WGC}^2$$

Both conditions are realised *only* for emergent heterotic string limits.

Failure of 4d tWGC $\Longleftrightarrow$ weak coupling limit decompactifies to

- defect gauge theory
- or strongly coupled higher-dim theory

EFT string limits

[Lanza,Marchesano,Martucci,Valenzuela'20-21], cf talk by L. Martucci

- At infinite distance in moduli space: axionic shift symmetries

  $N=1$ chiral multiplets $T_i = s_i + i a_i$: $T_i \sim T_i + i\,c$

- Dualisation of axions to 2-forms (in linear multiplets) is possible:

$$a_i \Longleftrightarrow B_2^i$$

EFT strings: strings charged under the 2-form fields:

$$S = \int_{\mathrm{string}} e_i\, B_{2i}^i + \ldots$$

- Backreaction of such strings induces in turn the infinite distance limit

$$T_i(z) = T_i^{(0)} - \frac{e_i}{2\pi} \log\left(\frac{z}{z_0}\right) \qquad z : \mathrm{transverse} \subset \mathbb{R}^{1,3}$$

- Those instantons are suppressed which are dual to the EFT string inducing the limit.

EFT string limits

N=1 Kähler moduli space in F-theory: [Lanza,Marchesano,Martucci,Valenzuela'20-21]

- Instantons: Euclidean D3 on effective divisors $D_i \in \mathrm{Eff}^1(B_3)$
- Strings: D3 on curves C in dual cone of movable curves $\mathrm{Mov}_1(B_3)$

EFT string limit:

For a subset $\mathcal{I} \subset \mathrm{Eff}^1(B_3)$ of generators of the effective cone:

$$\mathcal{V}_D \sim \lambda \to \infty\,, \quad \forall D \in \mathcal{I}\,, \qquad\qquad \mathcal{V}_{\hat{D}} < \infty \text{ for } \hat{D} \notin \mathcal{I}$$

primitive **EFT string**: $|\mathcal{I}| = 1$

Proposal: [Lanza,Marchesano,Martucci,Valenzuela'20-21]

primitive EFT string limit for generator D_i $\iff$ EFT string limit on dual curve C^i

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primitive EFT string limit for generator D_i $\Longleftrightarrow$ EFT string limit on dual curve C^i

Caveat: This might require change of chamber in Kähler moduli space (by flopping curves)

Note: Not all limits are EFT string limits (inhomogeneous scaling)
cf [Grimm,Lanza,Li'22]

(Quasi-)Primitive EFT Strings

For concreteness characterise limits in a fixed chamber:
How can one obtain a homogeneous scaling of divisor volumes?

Can scale independently the Kähler parameters v^i, i.e. the volumes of generators $\mathcal{C}^i$ of the Mori cone of effective curves

$$J = v^i J_i \,, \qquad v^i = \int_{\mathcal{C}^i} J$$

- Fix $\mathcal{C}^0$: generator of the Mori cone of effective curves
- Consider limit $\mathcal{V}_{\mathcal{C}^0} = \int_{\mathcal{C}^0} J = v^0 \to \infty$
- Co-scale other $v^i \to 0$ or $v^i \to \infty \implies$ minimal set $\mathcal{I}$ of generators of Eff^1 expanding homogeneously (such that no divisors scale to zero volume)

If this leads to a homogeneous scaling of divisors:
$\implies$ *quasi-primitive EFT string limit* (primitive if $|\mathcal{I}| = 1$)

Quasi-Primitive EFT Limits

Result: [Cota,Mininno,TW,Wiesner - to appear]

- Consider (quasi-)primitive EFT string limit for $\mathcal{C}^0$ dual to a Kähler cone generator J_0 (i.e. $v^0 \to \infty$ + co-scaling if needed).
- Associated EFT string obtained by wrapping a D3-brane on
  1. $C = \alpha\, J_0^2$ if $J_0^2 \neq 0$, or
  2. $C = \alpha\, J_0 \cdot J_i$ if $J_0^2 = 0$ with $J_i \neq J_0$ a suitable Kähler cone generator

Such C is in the cone $\text{Mov}_1(B_3)$, but not necessarily a generator (in given chamber)!

3 types of quasi-primitive EFT strings — classified by $q = 0, 1, 2$:
For curve $C = D_1 \cdot D_2$ define $Q_1 = D_1^2 \cdot D_2\,, \qquad Q_2 = D_1 \cdot D_2^2$

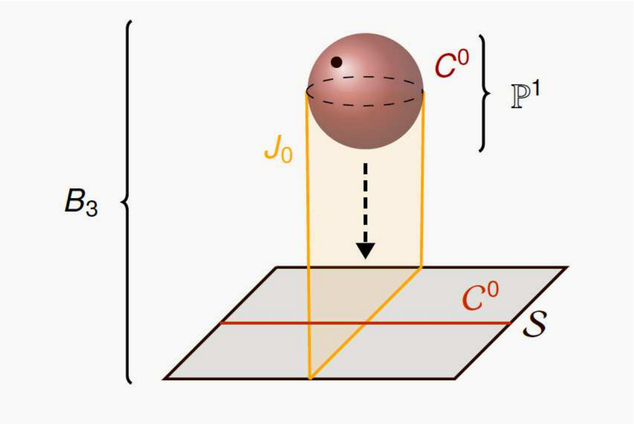
$$q(C) := \Theta(Q_1) + \Theta(Q_2)$$

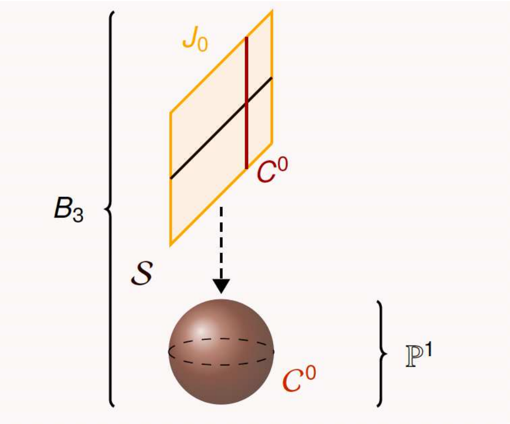
Quasi-Primitive EFT Limits

Geometric intuition:

$q = 0$:

- requires $J_0^3 = 0$
- $C = J_0^2$ (or $J_0 J_i$) is a $\mathbb{P}^1$ fiber
- always primitive:
  Emergent string limits!





$q = 1$:

- requires $J_0^2 = 0$
- $C = J_0 J_i$ lies inside surface fiber of B_3

$q = 2$:

- requires $J_0^3 \neq 0$
- $C = J_0^2$ is a general curve in homogeneously expanding B_3

Primitive Weak Coupling Limits

Focus first on primitive EFT limits - generalisations to quasi-primitive and more general limits later

Key result: [Cota,Mininno,TW,Wiesner - to appear]

Asymptotic factorisation of volume limit induced by a primitive string of type q:

$$\mathcal{V}_{B_3}^2 \to (\mathrm{Re}\, T_0)^{1+q} \, P_{2-q}(\mathrm{Re}\, T_{i\neq 0})$$

where

- $\mathrm{Re}\, T_0 \to \infty$ in primitive limit
- $\mathrm{Re}\,(T_{i\neq 0})$ finite

$\Longrightarrow$ Kähler potential:

$$K = -(1+q)\log(T_0 + \bar{T}_0) + \ldots$$

Primitive Weak Coupling Limits

Consider now gauge theory on 7-branes along divisor $\mathbf{S} = \kappa D_0 + \ldots$:

$$\frac{2\pi}{g_{\rm YM}^2} = \mathcal{V}_{\mathbf{S}} = \kappa \mathrm{Re}(T_0) + \ldots \to \infty$$

in a primitive limit $\mathrm{Re}(T_0) \to \infty$: D_0 dual to curve C^0 of type q

$$S_{4d} = \frac{M_{\rm Pl}^2}{2} \int \left[R \star 1 - (1+q) \frac{\mathrm{d}T_0 \wedge \star \mathrm{d}\bar{T}_0}{(T_0 + \bar{T}_0)^2} \right] - \frac{\kappa M_{\rm Pl}^2}{8} \int \left[\mathsf{Re}\, T_0 \, \mathsf{tr} |F|^2 - i \mathsf{Im}\, T_0 \, \mathsf{tr} F \wedge F \right] + \ldots .$$

✓ weak coupling limit for gauge theory

✓ weak coupling for primitive EFT string from D3-brane on q-curve C^0

$\Longrightarrow$ seems similar to weak coupling limit of a heterotic string $(q = 0)$

Natural question:

Is the tower WGC satisfied by the 'excitations' of the primitive EFT string?

cf. [Heidenreich,Reece,Rudelius'21] [Kaya,Rudelius'22]

Weak Gravity Conjecture

Worldsheet theory of string: $N = (0,2)$ with fermions charged under G

[Lawrie,Schäfer-Nameki,TW'16]; cf [Heidenreich,Reece,Rudelius'21]

Provided that the EFT string can be treated as a perturbative string:

- $M_k^2 = 8\pi T_{\text{EFT}}(n_k - E_0)$
- From elliptic genus/modularity: [Lee,Lerche,TW'18/19]

$$\text{max. charge-to-mass ratio}: \quad q_k^2 \geq 4mn_k \qquad m = \frac{1}{2}\mathbf{S}\cdot C^0$$

Exact WGC relation (Repulsive Force Conjecture [Palti'17]):

$$\frac{g_{\text{YM}}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\text{Pl}}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\text{Pl}}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\text{Pl}}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\text{Pl}}^2}\right)\right]$$

$$\textbf{Coulomb} \overset{!}{\geq} \textbf{Gravity} + \textbf{Yukawa}$$

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$$\frac{1}{1+q}\frac{1}{M_{\rm Pl}^2} \overset{!}{\geq} \frac{1+\frac{q}{2}}{1+q}\frac{1}{M_{\rm Pl}^2}$$

Satisfied only for $q=0$ EFT strings $\leftrightarrow$ critical heterotic string!

Weak Gravity Conjecture

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\rm Pl}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\rm Pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\right]$$

LHS:

$$\frac{2\pi}{g_{\rm YM}^2} = \mathcal{V}_{\mathbf{S}} = \kappa \mathrm{Re}(T_0) + \ldots\,, \qquad M_k^2 = 8\pi T_{\mathsf{EFT}}(n_k - E_0)$$

$$q_k^2 \geq 4 m n_k\,, \quad m = \frac{1}{2}\mathbf{S}\cdot C^0 = \kappa e_0\,, \quad \frac{T_{\mathsf{EFT}}}{M_{\rm Pl}^2} = e_0 L^0 = -\frac{e_0}{2}\frac{\partial K}{\partial \mathsf{Re}\, T_0} = \frac{e_0\,(1+q)}{2\,\mathsf{Re}\,T_0}$$

$$\Longrightarrow \frac{q_k^2 g_{\rm YM}^2}{M_k^2} = \frac{1}{1+q}\frac{1}{M_{\rm Pl}^2}$$

RHS:

$$g^{00} = -2\frac{\partial^2 K}{\partial(\mathsf{Re}\,T_0)^2} = \frac{2(1+q)}{(\mathsf{Re}\,T_0)^2} \Longrightarrow \frac{1}{4}\frac{M_{\rm Pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\rm Pl}^2}\right) = \frac{1}{2(1+q)}$$

$$\Longrightarrow \mathrm{RHS} = \frac{1+\frac{q}{2}}{1+q}\frac{1}{M_{\rm Pl}^2}$$

Weak Gravity Conjecture

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\rm Pl}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\rm Pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\right]$$

$$\frac{1}{1+q}\frac{1}{M_{\rm Pl}^2} \overset{!}{\geq} \frac{1+\frac{q}{2}}{1+q}\frac{1}{M_{\rm Pl}^2}$$

One *could* save it by *postulating* that

$$M_k^2 = 8\pi T_{\sf EFT}\mathfrak{n}(q)(n_k - E_0) \qquad \text{for} \quad \mathfrak{n}(q) = \frac{2}{2+q}$$

Instead we claim:

This is not the solution.

The tWGC is not asymptotically satisfied by EFT strings unless $q = 0$

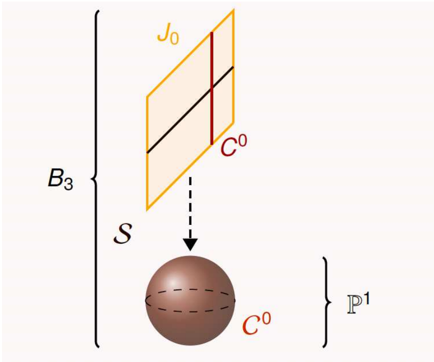
(critical heterotic string) – for good reason.

Interpretation of limits

Why does the tWGC relation for excitations of EFT string with $q \neq 0$ fail?

Quasi-primitive limits with $q \geq 1$ describe decompactification:

- q=1:
  decompactification to 6d gauge + gravity which generically is not weakly coupled



- q=2: decompactification to 10d, with 8d defect gauge theory

Reflected in EFT tension vs. species scale $\Lambda_{\text{sp,KK}}$ of KK tower

$\rightsquigarrow$ see talk by Mininno

$q = 0$	$q = 1$	$q = 2$
$\frac{T_{\text{EFT}}}{\Lambda_{\text{sp,KK}}} \ll \mathcal{O}(1)$	$\frac{T_{\text{EFT}}}{\Lambda_{\text{sp,KK}}} \sim \mathcal{O}(1)$	$\frac{T_{\text{EFT}}}{\Lambda_{\text{sp,KK}}} \gg \mathcal{O}(1)$
✓ 4d tWGC	6d tWGC *at best*	no tWGC possible

Generalisations

Result: [Cota,Mininno,TW,Wiesner - to appear]

- Conclusions carry over to general, non-EFT string limits
  $\rightsquigarrow$ see talk by Mininno
- Example: Combine q=1 quasi-primitive limit with weak coupling limit in 6d $\Longrightarrow$ 6d emergent string limits with tWGC

Asymptotic tWGC from string excitations requires

1. limit towards weak coupling of gauge theory and of the EFT string
2. species scale to be set by EFT string

$$\Lambda_{\rm sp} \stackrel{!}{=} \Lambda_{\rm sp,string} \sim T_{\rm string}$$

Both requirements together are only satisfied for $q = 0$ primitive EFT strings limits:

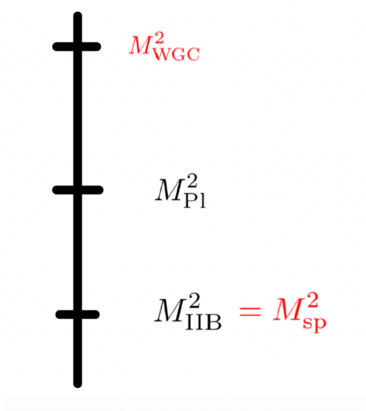
These are always the heterotic critical strings (i.e. emergent string limits)

Generalisations

In particular: No marginally super-extremal tower of states from open string excitations - even in weak coupling limits

Example: Type I string theory in 10d

$$M_{\rm WGC}^2 = g_{\rm YM}^2 M_{\rm Pl}^8 = \frac{1}{\sqrt{g_s}} M_{\rm Pl}^2$$
$$M_{\rm IIB}^2 = \sqrt{g_s} M_{\rm Pl}^2$$

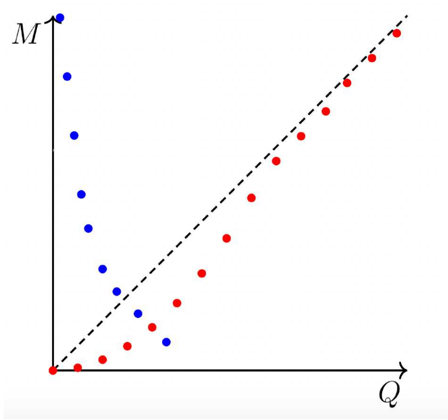


Species scale: [Dvali,Lüst '09] [Dvali, Gomez'09]

$M_{\rm sp} = M_{\rm IIB}$

Perturbative string states

$$\frac{g_{\rm YM}^2 Q_n^2}{M_n^2} \sim \frac{1}{g_s}\frac{1}{n}\frac{1}{M_{\rm Pl}^8} \overset{!}{\geq} \frac{1}{M_{\rm Pl}^8}$$



For fixed $g_s \ll 1$ only finite number of super-extremal states: $n < \frac{1}{g_s}$

Conclusions

Tower WGC works very differently for open string theories compared to closed strings:

1. 4d tWGC in limit $g_{\rm YM} \to 0$ only in emergent string limits
   $\Longleftrightarrow$ marginally super-extremal tower

2. Other weak coupling limits lead to decompactification:
   - Higher-dimensional tWGC requires higher-dim. emergent string limit
   - If defect theory, no tWGC expected nor realised.

3. Open string towers lead to finitely many highly super-extremal states (at fixed $g_s \ll 1$)
   - WGC satisfied, but not the tower version.
   - No immediate inconsistency with dimensional reduction.